Serie 1

1. Bestimmen Sie die Ordnung der folgenden partiellen Differentialgleichungen. Welche sind linear?

   (a) $u_{xy} \cdot u_x + u_{xxy} = 1$,

   (b) $u_x^2 + e^u \sin x = e^{u_y}$,

   (c) $tanx \cdot u_{xx} + e^y u_{xy} - \cos y = 0$,

   (d) $3u_{xy} + u_y^2 - e^x u - u_y^2 = 0$,

   (e) $5u_{xx} - 6u = 0$.

2. Show that each of the following equations has a solution of the form $u(x, y) = e^{\alpha x + \beta y}$ where $\alpha, \beta \in \mathbb{R}$. Find the constants α, β for each example.

   (a) $u_x + 3u_y + u = 0$,

   (b) $u_{xx} + u_{yy} = 5e^{x-2y}$,

   (c) $u_{xxxx} + u_{yyyy} + 2u_{xxyy} = 0$.

3. **(a)** Beweisen Sie, dass das System

$$\begin{aligned} u_x &= 2(x + y), \\ u_y &= 2x, \end{aligned}$$

   mit der Anfangsbedingung $u(0, 0) = 0$, genau eine Lösung hat.

   (b) Beweisen Sie, dass das System

$$\begin{aligned} u_x &= 2(x + y), \\ u_y &= Ax, \end{aligned}$$

   für $A \in \mathbb{R} \backslash \{2\}$, keine Lösung hat.

4. Read the section 1.4.4 of [PR], please derive the life expectancy for Random motion in a 3-dim cube D following the similar idea in the book.

   More precisely: Consider a particle in a 3-dim cube D. Divide the cubic into N^3 identical little cubes, and denote their vertices by $\{(x_i, y_i, z_i)\}$. The size of each edge of a small cube will be denote by δh. A particle located at an internal vertex (x_i, y_i, z_i) jumps during a time interval δt to one of its nearest neighbors with equal probabilty. When the particle reaches a boundary point it dies.

Question: What is the life expectancy $u(x,y,z)$ of a particle that starts its life at a point (x,y,z) in the limit

$$\delta h \to 0, \quad \delta t \to 0, \quad \frac{(\delta h)^2}{6\delta t} = k?$$

References

[PR] Y. Pinchover, J. Rubinstein, An introduction to Partial Differential Equations, Cambridge University Press(12. Mai 2005).